// CSES :- Two knights Problem

※ Solution

• Possible moves = total possible moves - moves where the knights attack each other.

※※ Total possible moves

By using Combination formula

$$nC_m = \frac{n!}{m!(n-m)!} \quad (1)$$

where

• $n \rightarrow k \times k$ chessboard

$m \rightarrow$ Two knights $\rightarrow 2$

Substitute

$$k^2C_2 = \frac{k^2!}{2!(k^2-2)!} \quad (2)$$

We can simplify this with the fact that if we substitute $m=2$ in equation (1)

$$nC_2 = \frac{n \cdot (n-1)\cancel{(n-2)!}}{2!\ \cancel{(n-2)!}}$$

$$= \frac{n^2-n}{2}$$

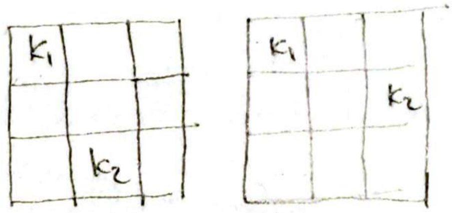
Substitute $n = k^2$

$$k^2C_2 = \frac{(k^2)^2 - k^2}{2} = \frac{k^4 - k^2}{2} \quad (3)$$

Σ knights counter-attack

- In chess, a knight attack in an L-shaped where it's either 3 squares horizontal and 1 square vertical or vice versa.

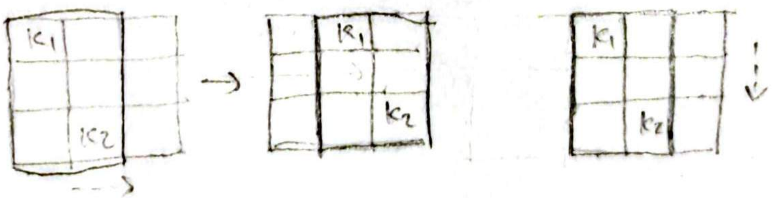
Example on a 3 × 3 board



- Based on the rule, we can see it as 2 × 3 matrix or 3 × 2 matrix where each has 2 valid moves.

Same goes for 3 × 3.

- Now, on a k × k board, we can imagine it as a window that can be sliGed, where both matrices could be slided horizontally and vertically.



In that sense, a 3 × 2 column matrix could have (k − 1) slide possible horizontally and (k − 2) vertically.

- The previous rule also applies on a 2 × 3 matrix except geometrially, it's (k − 2) horizontally and (k − 1) vertically.

(continuation from CSES - Two knights)

- Since each matrix has 2 possible moves,

$$\left.\begin{array}{l} \hookrightarrow 2\times3 \longrightarrow 2(k-1)(k-2) \\ \hookrightarrow 3\times2 \longrightarrow 2(k-1)(k-2) \end{array}\right\} 4(k-1)(k-2) \quad .. \quad (4)$$

- Put equation (3) and (4) together,

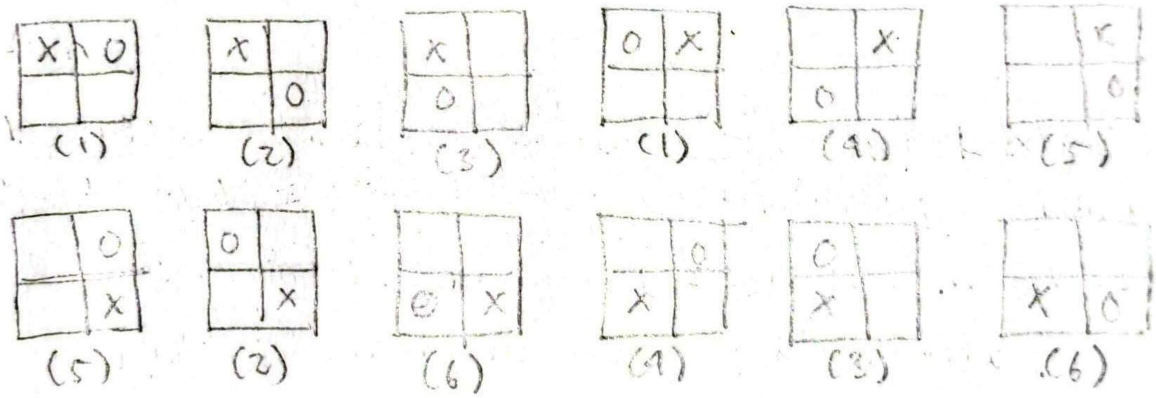
$$\Sigma \text{Moves} = \frac{k^4 - k^2}{2} - 4(k-1)(k-2)$$

*** Value testing

$$^{k^2}C_2 = \frac{k^4-k^2}{2}$$

For: $k=1 \rightarrow \frac{1^4-1^2}{2} = 0$

$k=2 \rightarrow \frac{2^4-2^2}{2} = \frac{16-4}{2} = 6$



Since order doesn't matter